Math 199 CD2: Linear Approximation

October 5, 2021

Remember, Δ = difference

Δy = difference in y

Newton's method is actually different.

1. Approximate the following quantity using ~~Newtons method~~ linear approximation, explitcitly describe what is f and a:

   (a) $\sqrt{16.2}$

   Let $f(x) = \sqrt{x}$, $x = 16$, $\Delta x = 0.2$

   $\Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$, $\Delta y = \sqrt{16.2} - \sqrt{16} = \sqrt{16.2} - 4$

   $\Rightarrow \sqrt{16.2} - 4 \approx \left(\frac{1}{2\sqrt{x}}\right) \cdot 0.2 = \frac{1}{8} \cdot \frac{2}{10} = \frac{1}{40}$

   $\Rightarrow \sqrt{16.2} \approx \frac{161}{40}$

   (b) $\sin(0.1)$

   Let $f(x) = \sin x$, $x = 0$, $\Delta x = 0.1$

   $\Rightarrow f'(x) = \cos(x)$. By appx rule:

   $\Delta y = \sin(0.1) - \sin(0) = \sin(0.1) = f'(x) \cdot \Delta x$

   $= \cos(0) \cdot 0.1 = 0.1$

   $\Rightarrow \sin(0.1) \approx 0.1$

   (c) $\sqrt[3]{124}$ $f(x) = \sqrt[3]{x} \Rightarrow f'(x) = \frac{1}{3} x^{-2/3}$

   $\Delta x = -1$, $x = 125$

   $\Rightarrow \Delta y = \sqrt[3]{124} - \sqrt[3]{125} = \sqrt[3]{124} - 5 = f'(x)\Delta x = \frac{1}{3} \cdot 125^{-2/3} \cdot \sqrt[3]{-1} = \ldots$

   (d) $\sin(\pi/3) = \frac{\sqrt{3}}{2}$

   Forget this one

2. A cubical box is to be built so that it holds 125 cubic inches. How precisely should the edge be made so that the volume will be correct to within 3 cubic inches?

$\Delta V = 3 \text{ inch}^3$, $r = 5$

$V = r^3 \Rightarrow \frac{dV}{dr} = 3r^2$

$\Rightarrow 3 = \frac{dV}{dr} \cdot \Delta r \Rightarrow 3 = 3 \cdot (5)^2 \cdot \Delta r$

$\Rightarrow \boxed{\Delta r = \frac{1}{25}}$

3. A solid steel cylinder has a radius of 2.5 cm and a height of 10 cm. A tight-fitting sleeve is to be made that will extend the radius to 2.6cm. Find the amount of steel needed for the sleeve.

$V = \pi r^2 h = 10\pi r^2$

let $r = 2.5$ & $\Delta r = 0.1$

$\Delta V = 10\pi (2.6)^2 - 10\pi (2.5)^2$

$\frac{dV}{dr} = 20\pi r \Rightarrow \frac{dV}{dr} \cdot \Delta r = 20\pi \cdot 2.5 \cdot 0.1$

$= 5\pi$

$\Rightarrow$ By appx. principle

$\Delta V = 5\pi + 10\pi (2.5)^2$