Math 199 CD2: Midterm 3 review

October 13, 2021

1 Related Rates

1. A Ferris wheel with a radius of 15 m is rotating at a rate of one revolution every two minutes. Exactly how fast (in m/min) is a rider rising when his seat is 27 m above ground level?

2. A 5-foot girl is walking toward a 20-foot lamppost at the rate of 6 feet per second. How fast is the tip of her shadow (cast by the lamp) moving?

3. A small funnel in the shape of a cone is being emptied of fluid at the rate of 12cubic centimeters per second. The height of the funnel is 20 centimeters and the radius of the top is 4 centimeters. How fast is the fluid level dropping when the level stands 5 centimeters above the vertex of the cone?

4. A particle moves on the hyperbola $x^2 - 18y^2 = 9$ in such a way that its y-coordinate increases at a constant rate of 9 units per second. How fast is its x-coordinate changing when $x = 9$?

2 Log/Exp differentiation

1. Find the derivative of the following function:

   (a) $y = x^x$

   (b) $y = (4x + 1)^{5x}$

(c) $y = x^{x^2+4}$

3 Rolle's Theorem, MVT, IVT

Please state all 3 theorem before you even attempt

Consider the polynomial $f(x) = 5x^3 - 2x^2 + 3x - 4$. Prove that $f(x)$ has a zero between 0 and 1 that is the only zero of $f(x)$.

4 Exponential Growth or Decay

1. A certain chemical decomposes exponentially. Assume that 200grams becomes 50 grams in 1hour. How much will remain after 3 hours?

2. If the world population in 1980 was 4.5 billion and if it is growing exponentially with a growth constant $K = 0.04 \ln 2$, find the population in the year 2030.

3. Fruit flies are being bred in an enclosure that can hold a maximum of 640 flies. If the flies multiply exponentially, with a growth constant $K = 0.05$ and where time is measured in days, how long will it take an initial population of 20 to fill the enclosure?

4. A bacterial culture, growing exponentially, increases from 100 to 400grams in 10 hours. How much was present after 3 hours?