Math 199 CD2: Review for Midterm 4

October 26, 2021

1 Finding Maxima, Minima and Inflection Points

1. Let $f(x) = \dfrac{x^6}{30} + \dfrac{x^5}{20} - x^4 + 3x + 20$ polynomial, sixth degree . Find all inflection points and intervals of concavity for f.

2. Find the absolute minimum and maximum of the function

$$f(x) = \ln(x) + \frac{2}{x}$$

defined on the interval $[1, e]$

3. Find the absolute maximum and minimum of $f(x) = 2\sin x + \sin 2x$ on $[0, 2\pi]$.

The following problems involved graph sketching. Show all relative extrema, inflection points, and asymptotes; indicate concavity; and suggest the behavior at infinity.

4. $\dfrac{x}{2x-1}$

5. $\dfrac{x^2}{x^2+1}$

6. Given the following information:

$$f(2) = 3, f'(2) = 0, f''(x) < 0, \forall x \in \mathbb{R}$$

What can you say about f?

2 L'Hopital Rule

1. $\lim\limits_{x \to 0} \dfrac{1}{x} - \dfrac{1}{\sin x}$

2. $\lim\limits_{x \to 0} \dfrac{1 - e^x}{x}$

3. $\lim\limits_{x \to +\infty} \dfrac{x^2}{(\ln x)^3}$

3 Exponentiate limit

1. $\lim\limits_{x \to \infty} \left(4e^{-3x} + 6\right)^{2e^{2x}}$

2. $\lim\limits_{x \to \infty} \left(e^{-x} + 6\right)^{e^{8x}}$

4 Approximation

Approximating the following value:

1. $\sqrt{26}$

2. $\sqrt{101}$

3. $f(x) = 5\ln(4x) + e^{x^4-1}$. Use linear approximation to evaluate $f(1.1)$, $f(0.9)$, $f(2.1)$ and $f(0.9)$

5   Error and Changing of Surface Area and Volume

1. If the side of a cube is measured with an error of at most 3 percent, estimate the percentage error in the volume of the cube.

2. A coat of paint of thickness 0.01 inch is applied to the faces of a cube whose edge is 10 inches, thereby producing a slightly larger cube. Estimate the number of cubic inches of paint used.

3. Assume, contrary to fact, that the earth is a perfect sphere, with a radius of 4000 miles. The volume of ice at the north and south poles is estimated to be about 8,000,000 cubic miles. If this ice were melted and if the resulting water were distributed uniformly over the globe, approximately what would be the depth of the added water at any point on the earth?

4. Measurement of the side of a cubical container yields the result 8.14cm, with a possible error of at most 0.005 cm. Give an estimate of the possible error in the value $V = (8.14)^3 = 539.35314 \ cm^3$ for the volume of the container.

6 MVT

True or False:

$$|1 - \sin(2x)| \leq 2|\frac{\pi}{4} - x|$$

for all x