Math 199 CD2: Limit and ϵ-δ

September 1, 2021

1. Let $E(h) = h^3$. We want to show $\lim_{h \to 0} E(h) = 0$.

   (a) If $\epsilon = 1$, then find δ so that when $0 < |h| < \delta$, we know $|E(h)| < \epsilon$.

   (b) If $\epsilon = \frac{1}{8}$, then find δ so that when $0 < |h| < \delta$, we know $|E(h)| < \epsilon$.

   (c) If we ϵ is not given explicitly, find δ in terms of ϵ so that when $0 < |h| < \delta$, we know $|E(h)| < \epsilon$.

2. For each of the following, use the ϵ-δ definition of the limit to show that the limit does not exist. Use words!

   (a) $\lim_{x \to 0} \frac{|x|}{x}$.

(b) $\lim_{x \to 1} \lfloor x \rfloor$ where $\lfloor x \rfloor$ is x rounded down to the nearest integer. For example, $\lfloor 1.7 \rfloor = \lfloor 1.2 \rfloor = 1$, $\lfloor -1/2 \rfloor = \lfloor -2/3 \rfloor = -1$

3. We want to show that $\lim_{x \to 2} (2 - 3x) = -4$.

(a) Fill in the blanks to set up the problem using a limit of zero at zero.
Let $E(h) = (2 - 3(2 + h)) - (-4) = -3h$. We say that _______ has limit _______ at _______ if
for every challenge number $\epsilon > 0$,
there is a response number $\delta > 0$ such that
if the input ____ is strictly between ____ and ____, but ____ is not equal to ____,
then the output _______ will be strictly between _______ and _______.

(b) Fill in the blanks to set up the problem using the traditional definition of the limit.
We say that _______ has limit _______ at _______ if
for every challenge number $\epsilon > 0$,
there is a response number $\delta > 0$ such that
if $0 <$ _______ $< \delta$,
then _______ $< \epsilon$.

(c) How are these two limit definitions the same? How are they different? Discuss with your group.

(d) Use either method to show $\lim_{x \to 2} (2 - 3x) = -4$.

4. Suppose $\lim_{x \to 0} E(x) = 0$. Use the ϵ-δ definition of a limit to prove the following:

(a) $\lim_{x \to 0} 2E(x) = 0$

(b) $\lim_{x \to 0} E(2x) = 0$