Math 199 CD2: More on Derivative

September 10, 2021

1. Find the derivative of the following functions:

   (a)
   $$f(x) = \left(\frac{x\sqrt[4]{x}}{x^{1/6}}\right)^{12} + \ln\left(\frac{\pi^3}{e^2+4}\right)$$

   $f(x) = \left(\frac{x^{5/4}}{x^{1/6}}\right)^{12} + \ln\left(\frac{\pi^3}{e^2+4}\right) = \frac{x^{15}}{x^2} + \text{const} = x^{13} + \text{const}.$

   $f'(x) = 13x^{12}$

   (b)
   $$f(x) = 8x^{-5/6} + \frac{10^{10}}{x^{-9/2}}$$

   $\frac{-5}{6} \cdot 8\, x^{-11/6} + \frac{9}{2} 10^{10} \cdot x^{7/2}$

   (c) $f(x) = (2x-3)^2$. Just some simple chain rule to get you started. But start with the product rule first and then see if product rule and chain rule would give a similar result

   $2(2x-3)$

2. At what points does the normal line (perpendicular line) to the curve $y = x^2 - 3x + 5$ at the point $(3, 5)$ intersect the curve?. Hint: the product of the slopes of 2 perpendicular lines should be -1. So find the formula for the normal line first

slope is $2x-3 \Rightarrow$ slope $= 3$ at $x=3$

$\Rightarrow$ slope of normal line is $\frac{-1}{3}$ (look at the hint for why)

$\Rightarrow$ equation for normal line is $y = \frac{-1}{3}x + b$. But $(3,5)$ is on the line for the normal line $\Rightarrow 5 = \frac{-1}{3} \cdot 3 + b \Rightarrow b = 6$

$\Rightarrow$ Equation is $y = \frac{-1}{3}x + 6$

Set $\frac{-1}{3}x + 6 = x^2 - 3x + 5$ to find the intersection

$\left(-\frac{1}{3}, \frac{55}{9}\right)$

3. Find the point(s) on the graph of $y = x^2$ at which the tangent line passes through $(2, -12)$

slope of tangent line is $2x$

2 points on the tangent line are (x, x^2), $(2, -12)$

$\Rightarrow$ The slope is $\frac{x^2+12}{x-2}$

$\Rightarrow 2x = \frac{x^2+12}{x-2} \Leftrightarrow (x-6)(x+2) = 0 \Rightarrow \begin{cases} x=6 \Rightarrow y=36 \\ x=-2 \Rightarrow y=4 \end{cases}$

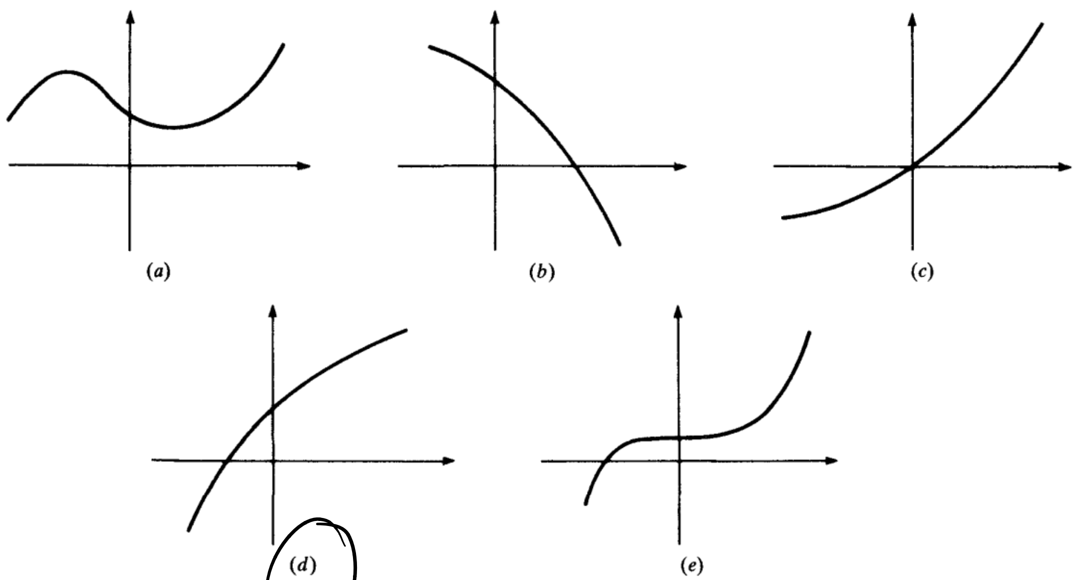
4. Find the point(s) on the graph of $y = x^2$ at which the tangent line is parallel to the line $y = 6x - 1$. Hint: 2 lines are parallel must have the same slope

Slope has to be the same if $\parallel$

$\Rightarrow 2x = 6 \Rightarrow x = 3$

$\Rightarrow (3, 9)$

5. We haven't talked much about the second derivative but it's super useful for visualizing the function. The second derivative tells you whether the function is concave up (if positive) or down (if negative) at a certain point.
If, for all x, $f'(x) > 0$ and $f''(x) < 0$, which of the curves below could be part of the graph of f?



only one concave down & ↑ function